



# Quasiparticle spectrum and optical properties of SnO<sub>2</sub>

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ABINIT Developer Workshop  
Han-sur-Lesse, April 11-14, 2011



## Outline

- Introduction
- Electronic structure
- Optical absorption
- Quasiparticle spectrum of  $\text{SnO}_2$
- Pressure effects
- Scalar relativistic pseudopotential
- Optical properties
- Conclusions and outlook



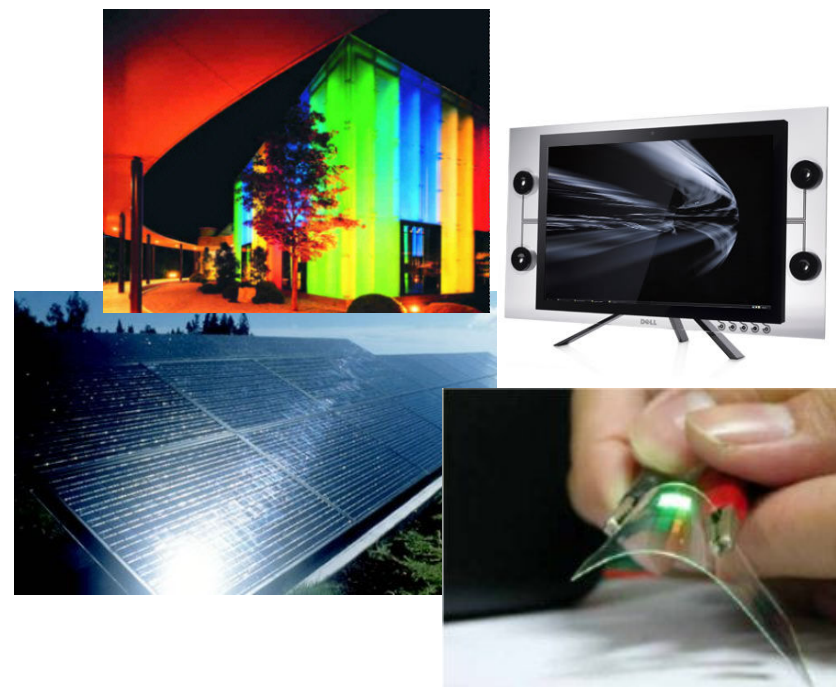
## Introduction

Transparent  
conducting oxides:

- band gap  $\geq 2$  eV
- resistivity  $\leq 10^{-4}$   $\Omega$  cm  
and  $\mu \geq 50$   $\text{cm}^2/\text{V s}$

$\text{SnO}_2$ :

- band gap  $\sim 3.6$  eV<sup>1</sup>
- Ta-doped<sup>2</sup> resistivity  
 $\sim 2 \times 10^{-4}$   $\Omega$  cm ...  
and  $\mu \sim 60$   $\text{cm}^2/\text{V s}$



<sup>1</sup>M. Nagasawa and S. Shionoya, Phys. Lett. 22, 409 (1966).

<sup>2</sup>S. Nakao *et al.*, Appl. Phys. Express 3, 031102 (2010)



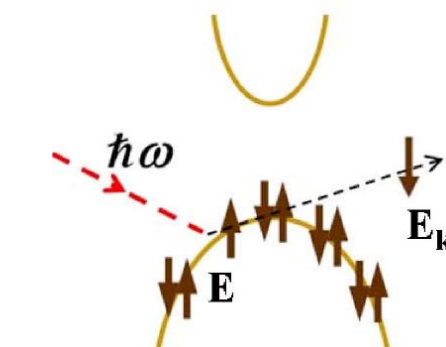
# Electronic structure

The electron Green function describes the evolution of an excitation

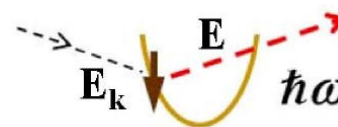
$$G(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2) = -\frac{i}{\hbar} \langle \Psi_0 | T[\hat{\psi}(\mathbf{r}_1, t_1) \hat{\psi}^\dagger(\mathbf{r}_2, t_2)] | \Psi_0 \rangle$$

The Green function poles are single-particle excitation energies

$$G(\mathbf{r}_1, \mathbf{r}_2; \omega) = \frac{1}{\hbar} \sum_i \frac{\langle \Psi_0^{(N)} | \hat{\psi}(\mathbf{r}) | \Psi_i^{(N+1)} \rangle \langle \Psi_i^{(N+1)} | \hat{\psi}^\dagger(\mathbf{r}') | \Psi_0^{(N)} \rangle}{\hbar\omega - \epsilon_i^{(N+1)} + i\eta} + \frac{1}{\hbar} \sum_i \frac{\langle \Psi_0^{(N)} | \hat{\psi}^\dagger(\mathbf{r}') | \Psi_i^{(N-1)} \rangle \langle \Psi_i^{(N-1)} | \hat{\psi}(\mathbf{r}) | \Psi_0^{(N)} \rangle}{\hbar\omega - \epsilon_i^{(N-1)} - i\eta}$$



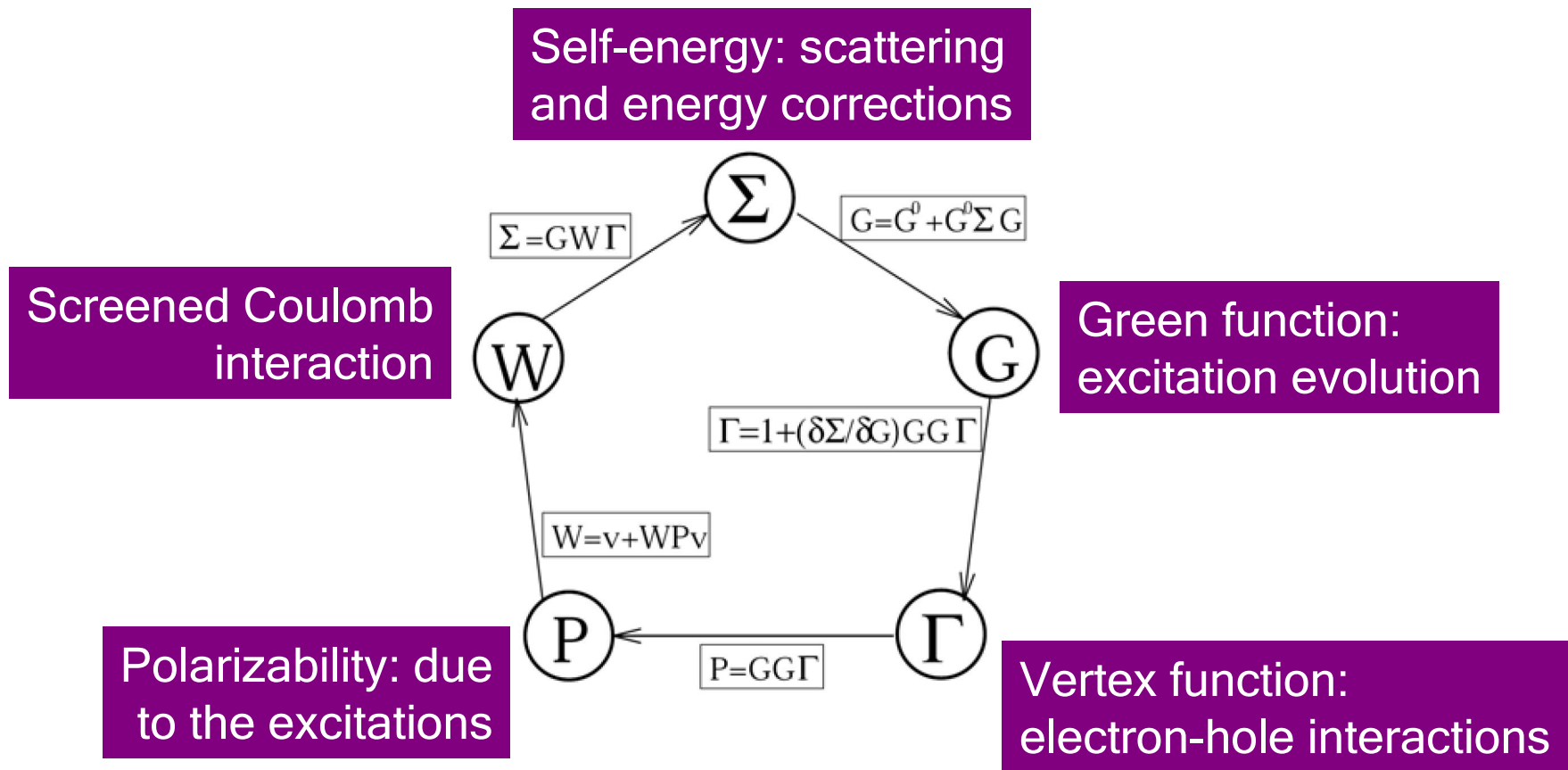
Photoemission:  $N \rightarrow N-1$



Inverse photoemission:  
 $N \rightarrow N+1$



# Hedin equations & GW approximation<sup>1,2</sup>



<sup>1</sup>L. Hedin, Phys. Rev. **139**, A796 (1965).

<sup>2</sup>W. G. Aulbur, L. Jönsson, and J. W. Wilkins, Solid State Phys. **54**, 1 (1999).



# GW scheme

From the Dyson equation

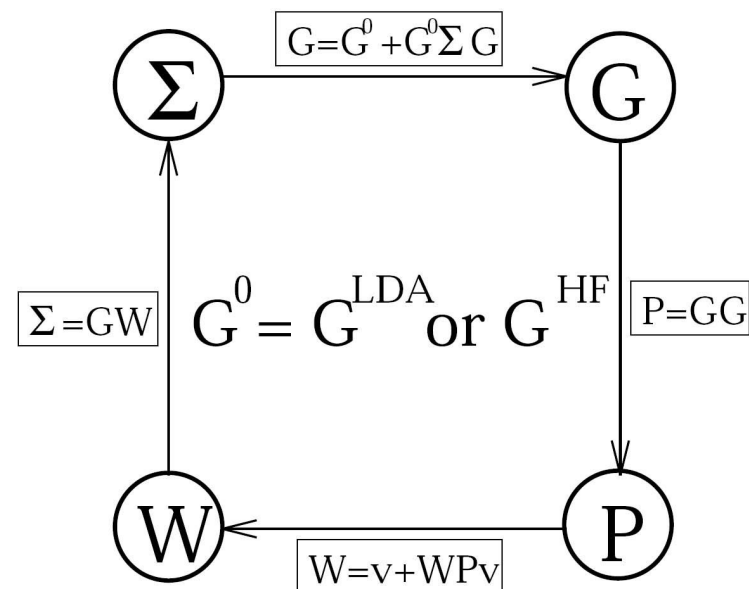
$$\mathcal{H}_0(\mathbf{r})\psi_j(\mathbf{r}) + \int d^3r' \hbar\Sigma^*(\mathbf{r}, \mathbf{r}'; \epsilon_j)\psi_j(\mathbf{r}') = \epsilon_j\psi_j(\mathbf{r}).$$

DFT-local density approx.

$$\Sigma(\mathbf{r}, \mathbf{r}'; \omega) = \delta(\mathbf{r} - \mathbf{r}')V_{xc}(\mathbf{r})$$

“GW” approximation

$$\Gamma = 1 \Rightarrow \Sigma = GW$$



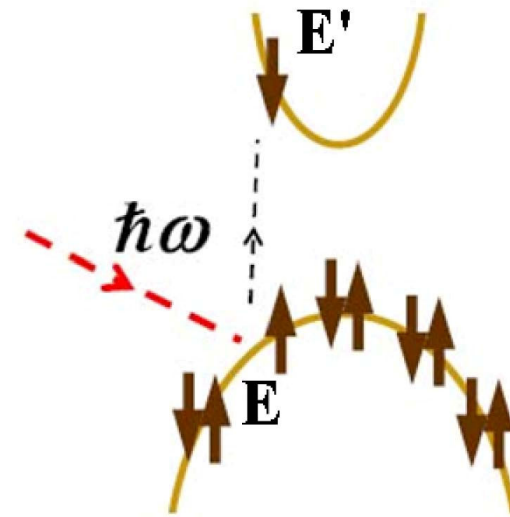


# Optical absorption

- Macroscopic dielectric function

$$\begin{aligned}\varepsilon_M(\omega) &\equiv \lim_{\mathbf{q} \rightarrow 0} \frac{1}{\varepsilon_{\mathbf{G}=0, \mathbf{G}'=0}^{-1}(\mathbf{q}, \omega)}, \\ &= 1 - \lim_{\mathbf{q} \rightarrow 0} [v(\mathbf{q})_0 \bar{P}_{\mathbf{G}=\mathbf{G}'=0}(\mathbf{q}, \omega)]\end{aligned}$$

Photoabsorption: two-particle  
excitation process ...  
but the GW polarizability does not  
contain electron-hole interactions!

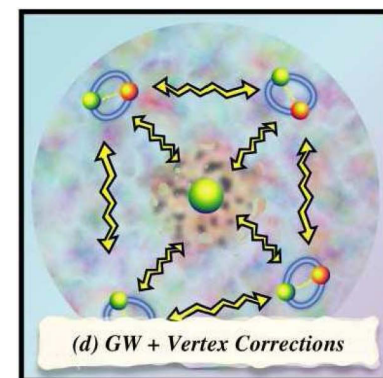




## • Bethe-Salpeter

Vertex corrections:

$$\frac{\delta\Sigma(12)}{\delta G(45)} \simeq i\hbar\delta(14)\delta(25)W(1+2)$$



=> Bethe-Salpeter equation for the polarisability

$${}^4\bar{P} = {}^4P_{IQP} + {}^4P_{IQP}K {}^4\bar{P},$$

$$K(1234) = \delta(12)\delta(34)\bar{v}(13) - \delta(13)\delta(24)W(12)$$

${}^4P$  = “4-point” polarisability

${}^4P_{IQP}$  = independent quasiparticle polarisability

$v$  = bare Coulomb interaction

$W$  = dynamically screened Coulomb interaction





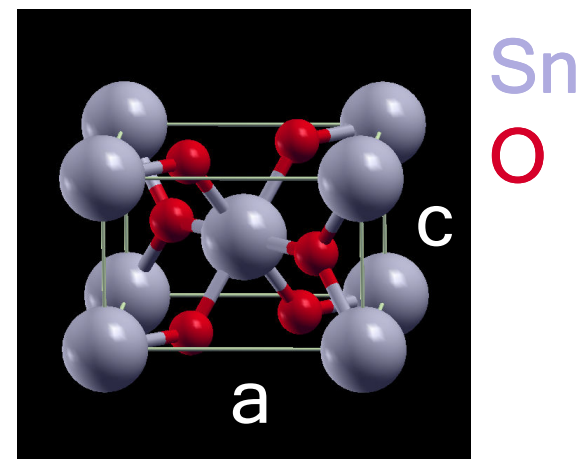
# Quasiparticle spectrum of SnO<sub>2</sub>

- DFT calculations

22e PP required: [Ge]4s<sup>2</sup>4p<sup>6</sup>4d<sup>10</sup>5p<sup>2</sup>5s<sup>2</sup>  
(Opium code<sup>1</sup>) ... 105 Ha ecut!

## Structural properties

|                   | a (Å)  | c (Å)  | u       | B (GPa)              |
|-------------------|--------|--------|---------|----------------------|
| Exp. <sup>2</sup> | 4.7374 | 3.1864 | 0.30562 | 212.3 <sup>(3)</sup> |
| Theory            | 4.7154 | 3.1864 | 0.30605 | 211.7                |



SnO<sub>2</sub> structure:  
Rutile (*P4<sub>2</sub>mm*)

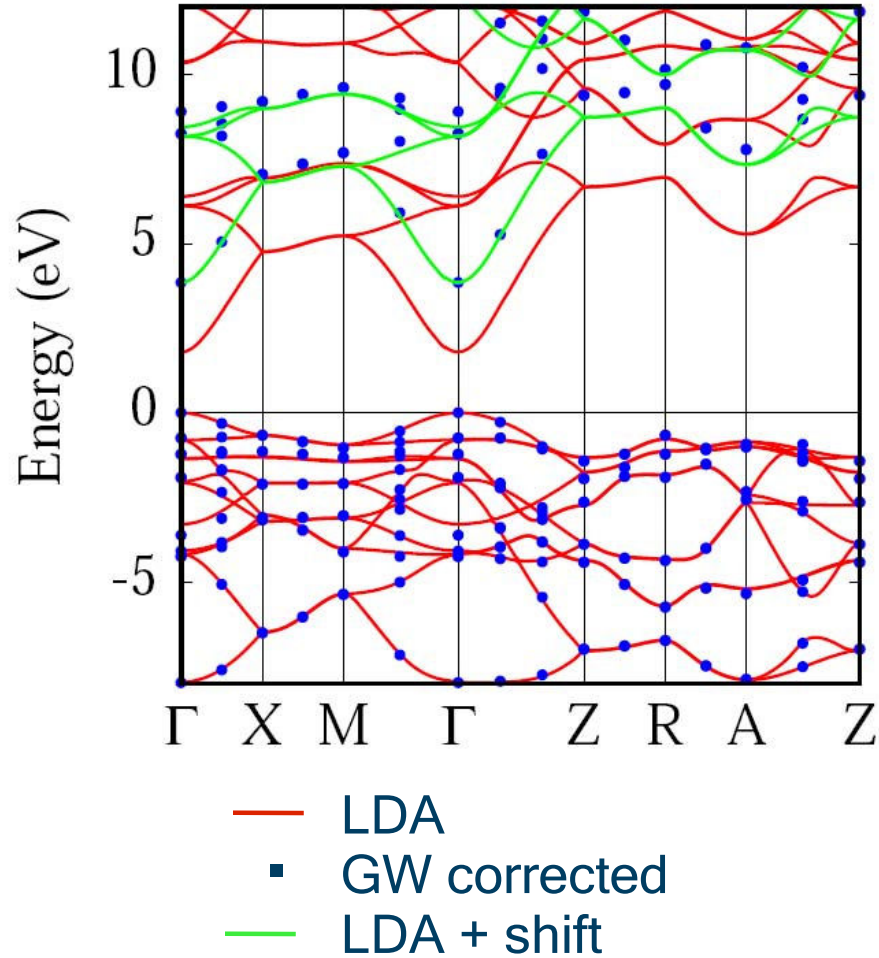
<sup>1</sup>[opium.sourceforge.net/index.html](http://opium.sourceforge.net/index.html)

<sup>2</sup>A. Bolzan *et al.*, Acta Cryst. B53, 373 (1997).

<sup>3</sup>E. Chang and E. Graham, J. Geophys. Res. 80, 2595 (1975).



## • Electronic structure



For ref., optical gap at  $\Gamma$ : 3.6 eV

Gap values (eV):

| k pt       | $\Gamma$ | X    | M    | Z     |
|------------|----------|------|------|-------|
| LDA        | 1.80     | 5.42 | 6.17 | 7.99  |
| GW         | 3.85     | 7.69 | 8.72 | 10.81 |
| $\Delta E$ | 2.05     | 2.27 | 2.55 | 2.82  |

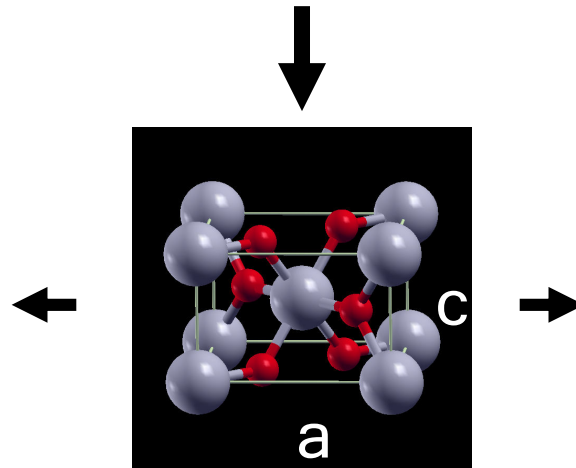
Effective masses ( $m_e$ ):

|                    | $m_{\perp}^*$ | $m_{\parallel}^*$ | $m_p^*$ |
|--------------------|---------------|-------------------|---------|
| Exp <sup>1</sup> . | 0.299         | 0.234             | 0.275   |
| Theory             | 0.253         | 0.223             | 0.271   |

<sup>1</sup>K. Button et al., Phys. Rev. B 4, 4539 (1971).

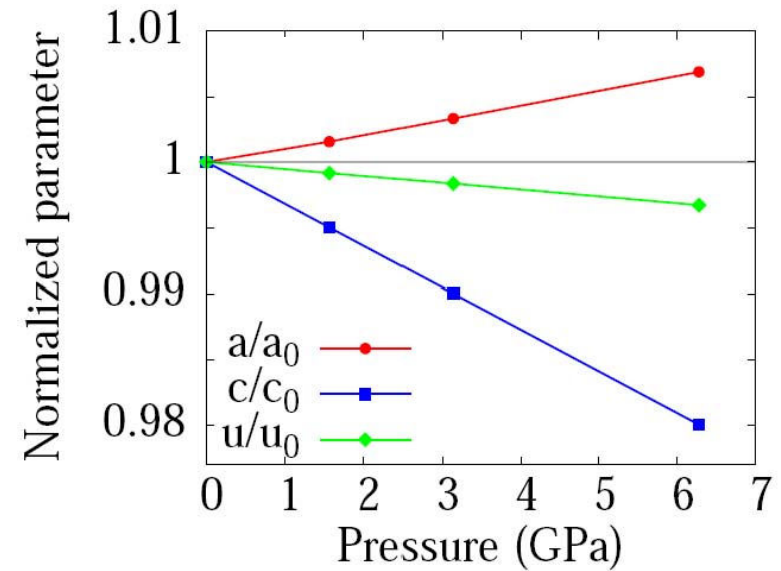


Applied uniaxial pressure



“anaddb”  
(35 Gb storage)

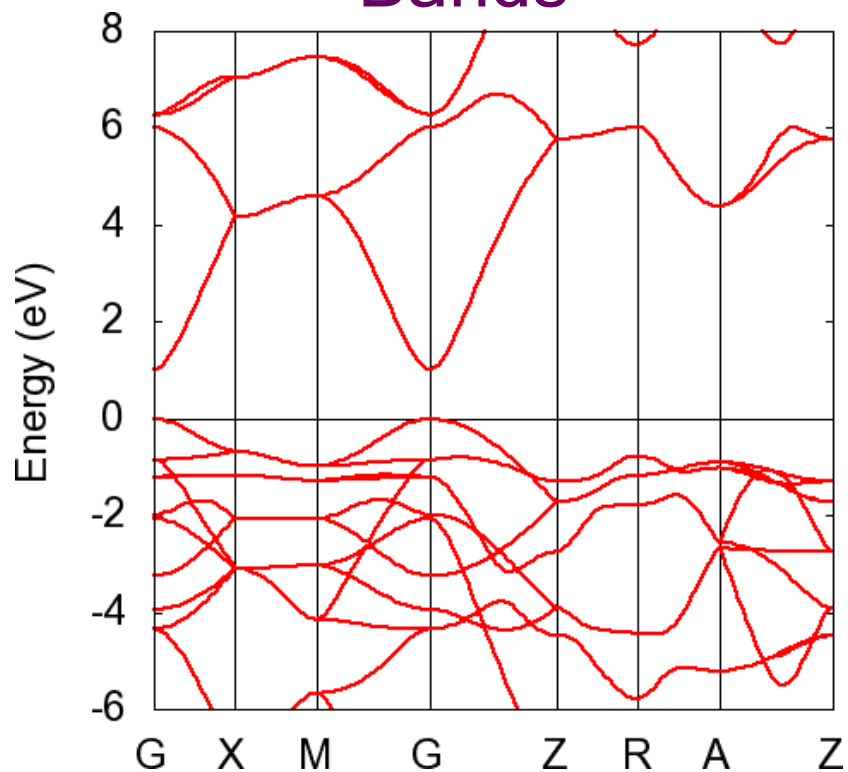
## Pressure effects



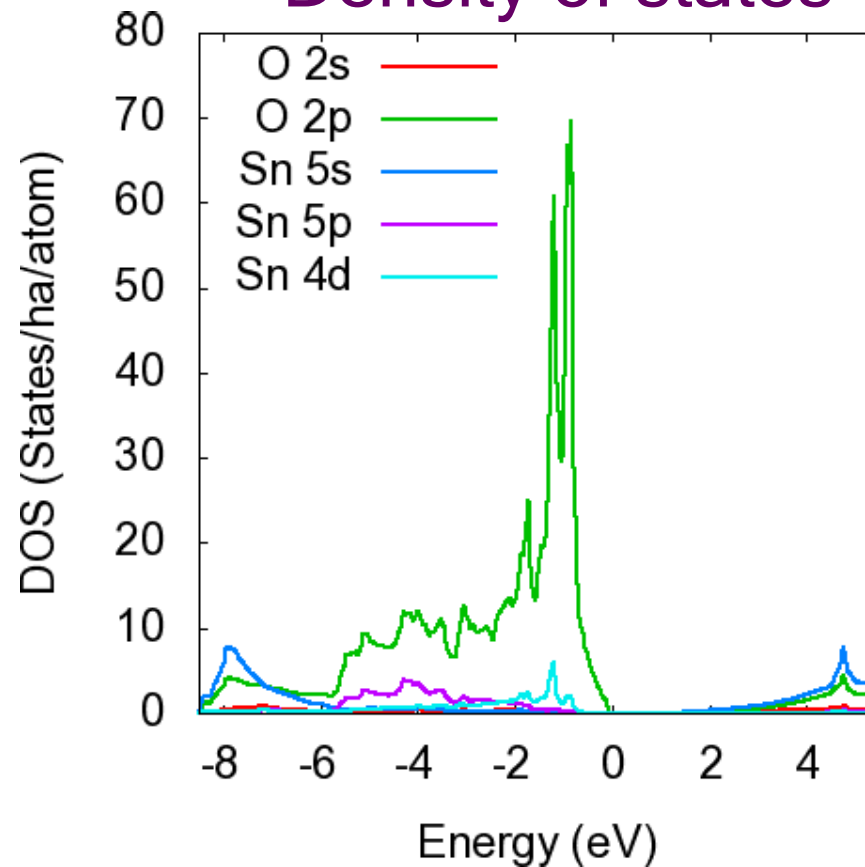
|                   | $C_{11}$ | $C_{33}$ | $C_{12}$ | $C_{13}$ | $C_{44}$ | $C_{66}$ | GPa               |
|-------------------|----------|----------|----------|----------|----------|----------|-------------------|
| Exp. <sup>1</sup> | 261.7    | 449.6    | 177.2    | 155.5    | 103.1    | 207.4    |                   |
| Theory            | 274.3    | 412.5    | 180.9    | 149.8    | 94.3     | 202.9    |                   |
|                   | $S_{11}$ | $S_{33}$ | $S_{12}$ | $S_{13}$ | $S_{44}$ | $S_{66}$ | TPa <sup>-1</sup> |
| Theory            | 6.801    | 3.186    | -3.913   | -1.049   | 10.608   | 4.929    |                   |



## Bands



## Density of states

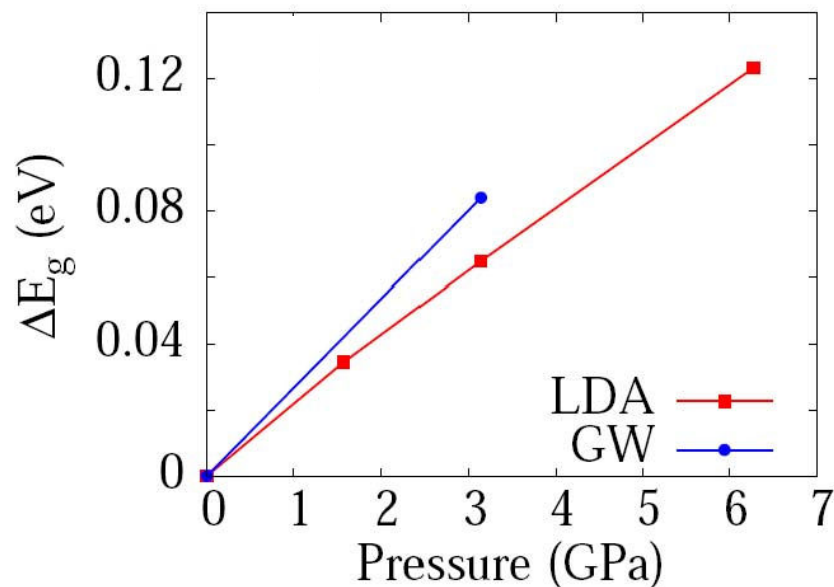




## Pressure and GW

GW corrections at high symmetry k-points

| k pt                   | $\Gamma$ | X    | M    | Z    |
|------------------------|----------|------|------|------|
| $\Delta E$ (P=0)       | 2.05     | 2.27 | 2.55 | 2.82 |
| $\Delta E$ (P=3.1 GPa) | 2.08     | 2.30 | 2.57 | 2.84 |



$$P_{\text{coeff GW}} = 27 \text{ meV/GPa}$$

$$P_{\text{coeff GW}} / P_{\text{coeff LDA}} = 0.74$$



# Scalar relativistic pseudopotential

Scalar relativistic effects

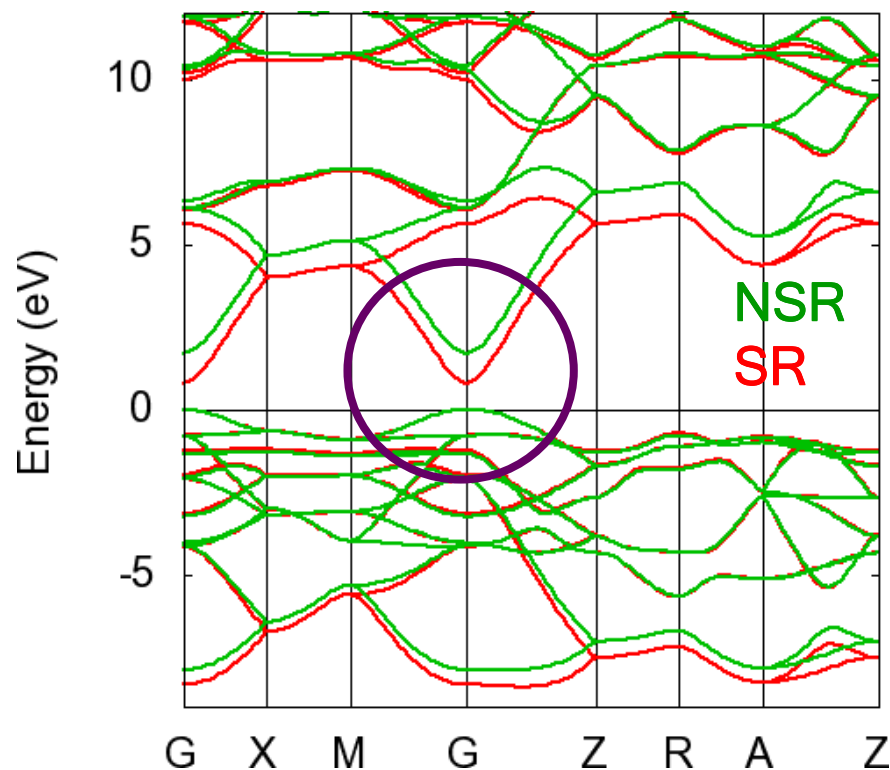
⇒ shrinkage of core s and p shells

⇒ change of bandwidths and  $E_g$

Structural parameters

|                   | a (Å)  | c (Å)  | u       |
|-------------------|--------|--------|---------|
| Exp. <sup>2</sup> | 4.7374 | 3.1864 | 0.30562 |
| Theory            | 4.7185 | 3.1849 | 0.30620 |

SR  $E_g = 0.81$  eV  
(vs 1.80 eV in NSR !)





## Elastic constants

|                   | $C_{11}$ | $C_{33}$ | $C_{12}$ | $C_{13}$ | $C_{44}$ | $C_{66}$ |
|-------------------|----------|----------|----------|----------|----------|----------|
| Exp. <sup>1</sup> | 261.7    | 449.6    | 177.2    | 155.5    | 103.1    | 207.4    |
| Theory            | 238.7    | 416.9    | 177.4    | 152.8    | 91.6     | 204.4    |
|                   | $S_{11}$ | $S_{33}$ | $S_{12}$ | $S_{13}$ | $S_{44}$ | $S_{66}$ |
| Theory            | 9.808    | 3.281    | -6.520   | -1.205   | 10.916   | 4.892    |

$$B_{SR} = 202.6 \text{ GPa}$$

$$B_{NSR} = 211.7 \text{ GPa}$$

$$B_{exp} = 212.3 \text{ GPa}$$

## GW gaps (eV)

| k pt | $\Gamma$ | X    | M    | Z     |
|------|----------|------|------|-------|
| NSR  | 3.85     | 7.69 | 8.72 | 10.81 |
| SR   | 2.65     | 6.75 | 7.59 | 9.53  |

Gaps go down

## Effective masses ( $m_e$ )

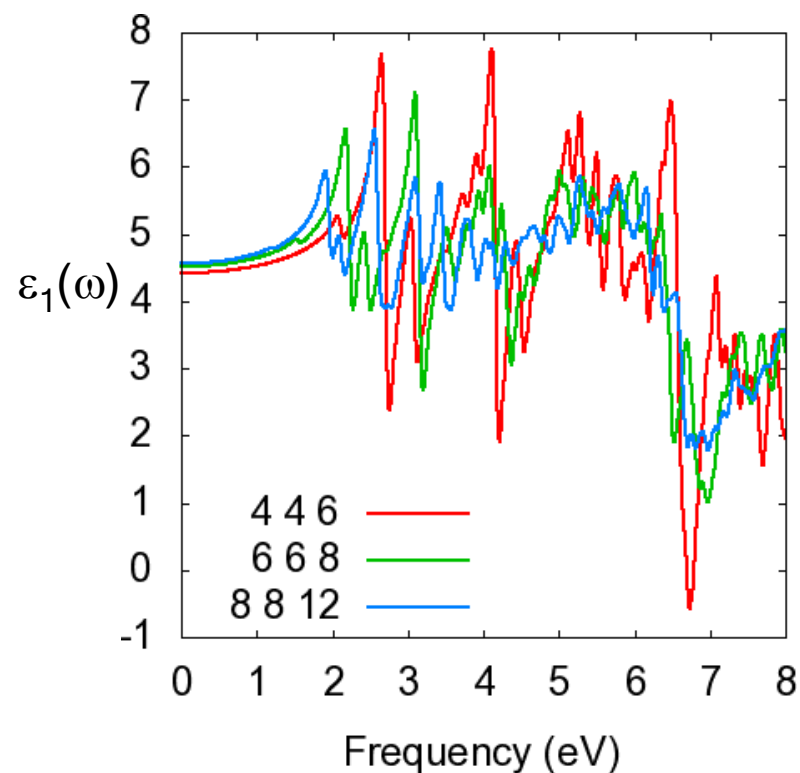
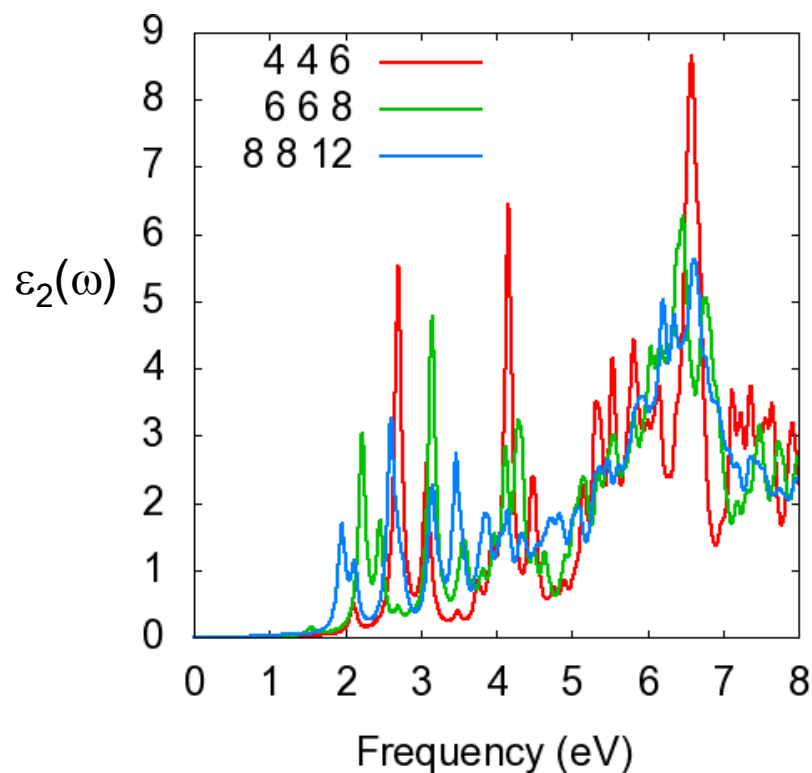
|     | $m_{\perp}^*$ | $m_{\parallel}^*$ | $m_p^*$ |
|-----|---------------|-------------------|---------|
| NSR | 0.253         | 0.223             | 0.271   |
| SR  | 0.189         | 0.170             | 0.204   |

... and effective masses go down



# Optical properties

“optic” code

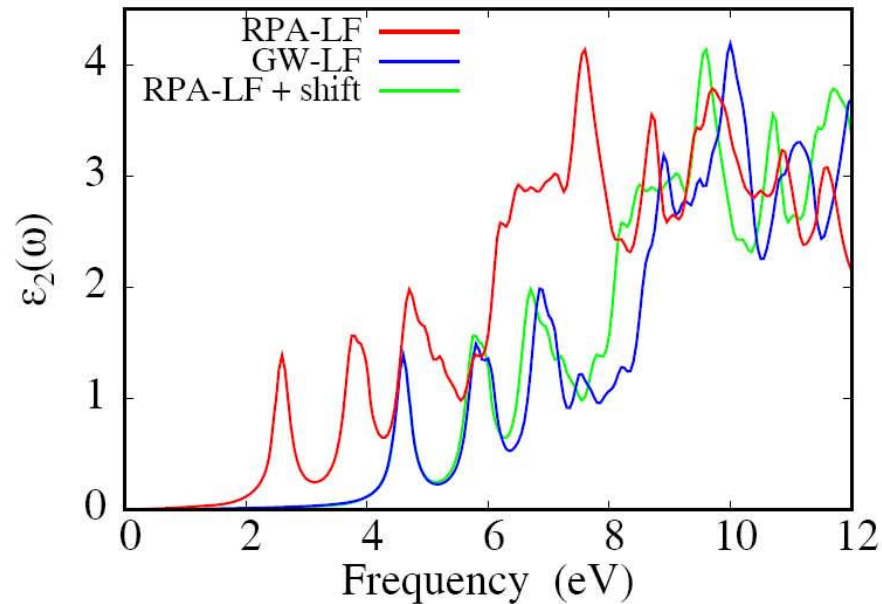


Caution: dense k-point needed (> 5Gb memory/proc.)

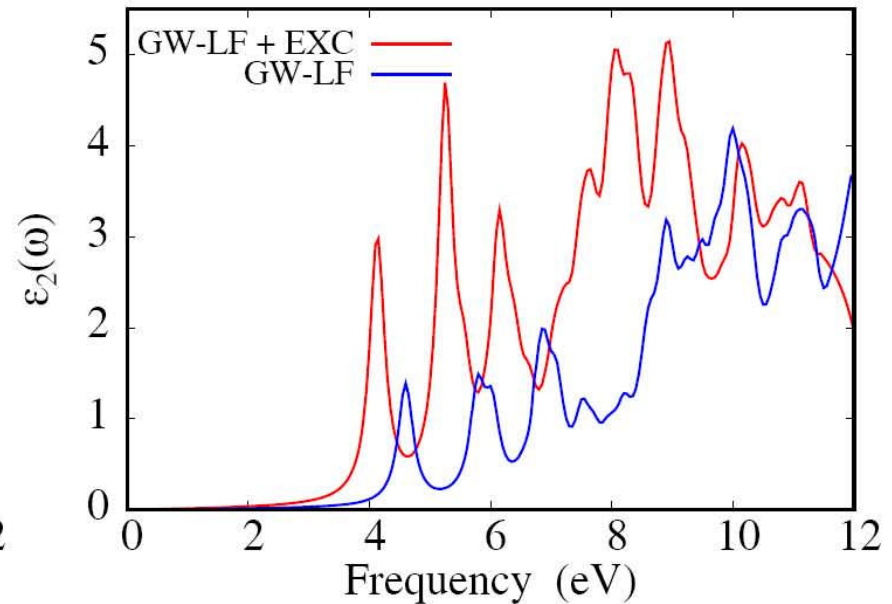




## Imaginary part of the macroscopic dielectric function: LDA-RPA vs GW-RPA vs Bethe-Salpeter



Absorption edge and structure at higher energies



Absorption edge and structure also at low energies



## Conclusions and outlook

- Structural and elastic properties are accurately predicted.
- GW corrections are band and momentum dependent.
- ... but require more work: quasi-particle self-consistent calculation (cassiterite\_i\_SCR = 65 Gb, GW corrections = 11 Gb/proc. with spectral method).
- Bethe-Salpeter shows important excitonic effects. Future work: Wannier interpolation for GW eigenvalues and LDA for wavefunctions?