

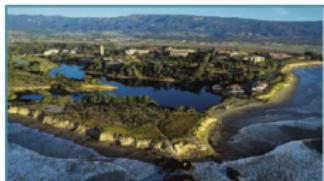
# Issues with simulations of non-collinear magnets in DFT

**Eric Bousquet and Nicola Spaldin**

Materials Theory, ETH Zürich

[eric.bousquet@mat.ethz.ch](mailto:eric.bousquet@mat.ethz.ch)

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Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich

# DFT and non-collinear magnetism

General density within non collinear magnetism:

$$\rho = \begin{pmatrix} \rho^{\uparrow\uparrow} & \rho^{\uparrow\downarrow} \\ \rho^{\downarrow\uparrow} & \rho^{\downarrow\downarrow} \end{pmatrix} = \begin{pmatrix} n + m_z & m_x - im_y \\ m_x + im_y & n - m_z \end{pmatrix}$$

$n$  = electronic density ;  $m$  = magnetization density

Need to take into account the Spin-Orbit coupling.

Potential:

$$V = \begin{pmatrix} V^{\uparrow\uparrow} & V^{\uparrow\downarrow} \\ V^{\downarrow\uparrow} & V^{\downarrow\downarrow} \end{pmatrix}$$

# Non-collinear magnetism and projects

- Non-collinear magnets:
  - Spin Canting
  - Spin Spiral
  - ...
- Magnetocrystalline Anisotropy:
  - Magnetostriiction
  - Magnetoelastic coef
  - ...
- Response to magnetic/electric field:
  - Magnetic/Electric susceptibility
  - Magnetoelectric response
  - Ferrotoroidal response
- Get these responses with DFPT

# Localised orbitals: LSDA+ $U$

In the collinear scheme  
LSDA+ $U$  Liechtenstein:

$$V_{LSDA+U}^{\sigma} = V_{LSDA}^{\sigma} + V_{Coulomb}^{\sigma} - V_{double-count}^{\sigma}$$

$$V_{Coulomb}^{\sigma} = V_U n^{-\sigma} + (V_U - V_J) n^{\sigma}$$

$$V_{double-count}^{\sigma} = -U \left( N - \frac{1}{2} \right) + \frac{1}{2} J (N^{\sigma} - 1) \quad .$$

(FLL double counting)

# LSDA+ $U$ and non-collinear magnetism

LSDA+ $U$  Liechtenstein:

Diagonal terms:

$$\sigma = \uparrow\uparrow, \downarrow\downarrow \quad V_{Coulomb}^{\sigma} = V_U n^{-\sigma} + (V_U - V_J) n^{\sigma}$$

$$\sigma = \uparrow\uparrow, \downarrow\downarrow \quad V_{double-count}^{\sigma} = -U\left(N - \frac{1}{2}\right) + \frac{1}{2}J(N-1) \pm \frac{1}{2}Jm_z$$

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Off-diagonal terms:

$$\sigma = \uparrow\downarrow, \downarrow\uparrow \quad V_{Coulomb}^{\sigma} = -V_J n^{\sigma}$$

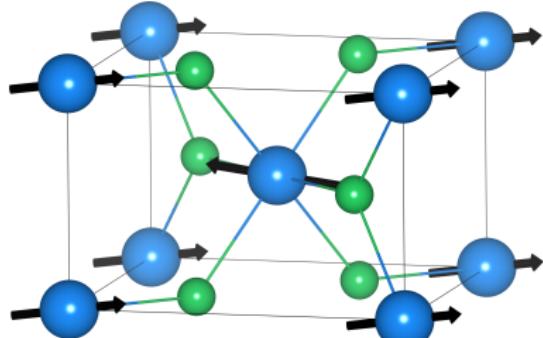
$$\sigma = \uparrow\downarrow, \downarrow\uparrow \quad V_{double-count}^{\sigma} = \frac{1}{2}J(m_x \pm im_y)$$

LSDA+ $U$  Dudarev = Liechtenstein with  $J=0$

→ No correction for  $\sigma = \uparrow\downarrow, \downarrow\uparrow$  !

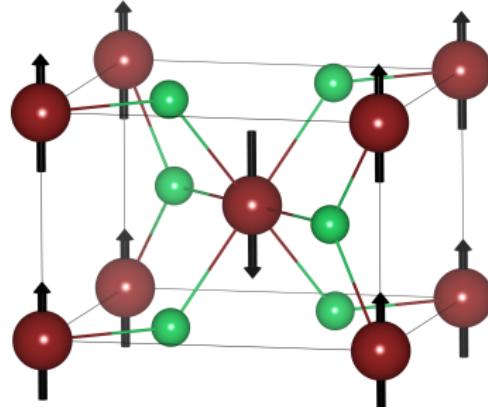
# Magnetocrystalline anisotropy: Difluorites

$\text{NiF}_2$



Spins in-plane,  
canting out-of-plane

$\text{MnF}_2, \text{CoF}_2, \text{FeF}_2$

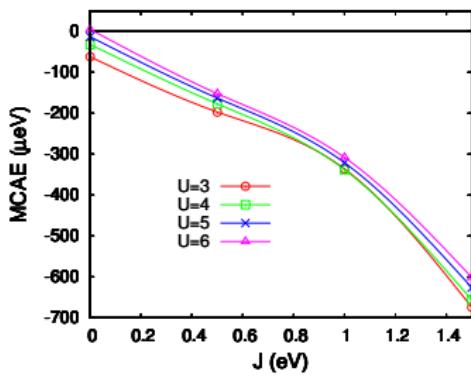


Spins out-of-plane,  
no canting

$$\text{MCAE} = E(\text{in-plane}) - E(\text{out-of-plane})$$

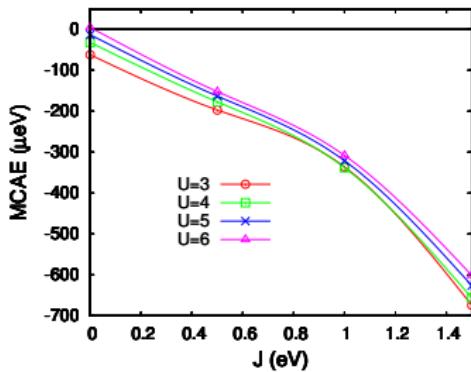
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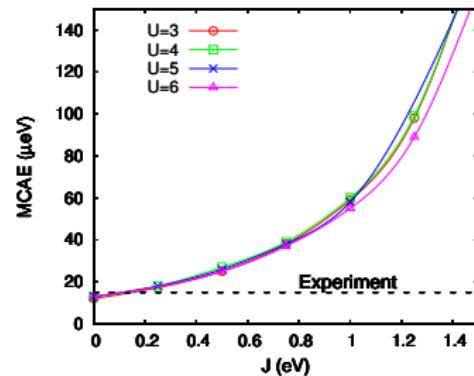


# Magnetocrystalline anisotropy: Difluorites

NiF<sub>2</sub>

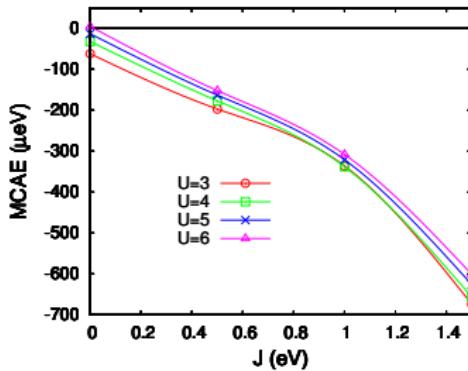


MnF<sub>2</sub>

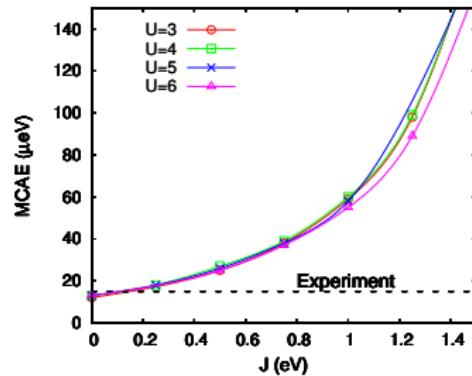


# Magnetocrystalline anisotropy: Difluorites

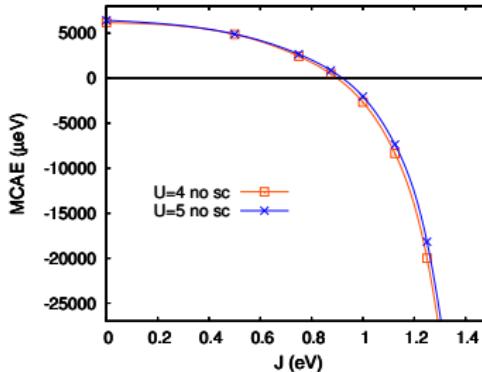
NiF<sub>2</sub>



MnF<sub>2</sub>

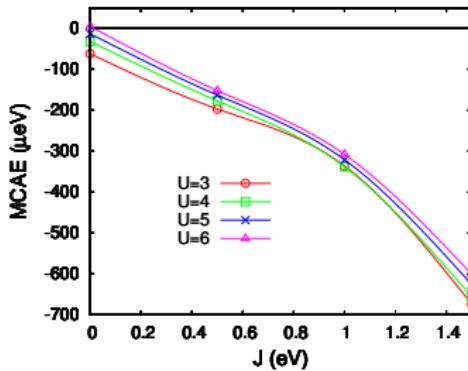


CoF<sub>2</sub>

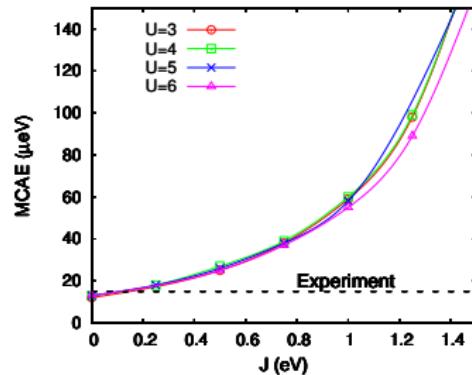


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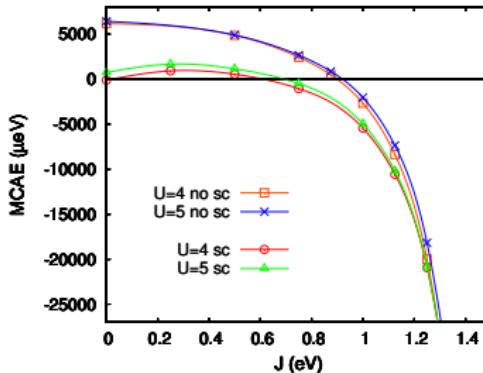
NiF<sub>2</sub>



MnF<sub>2</sub>

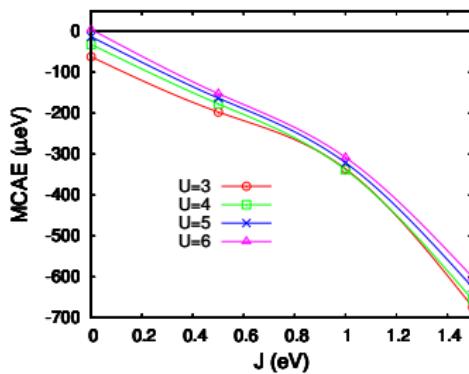


CoF<sub>2</sub>

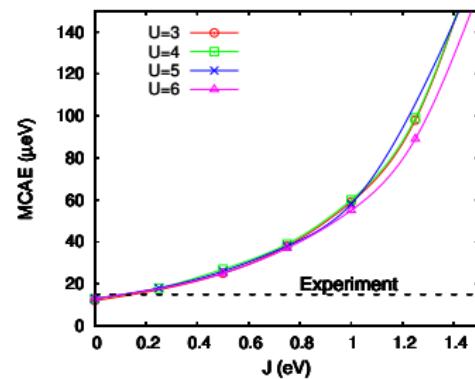


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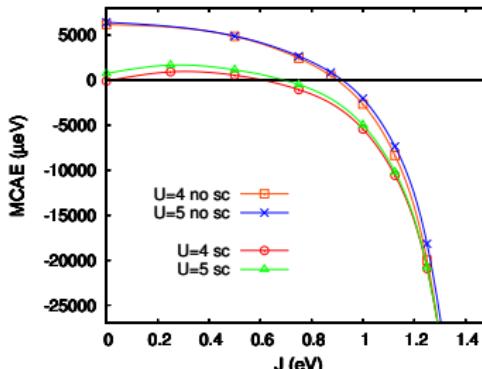
NiF<sub>2</sub>



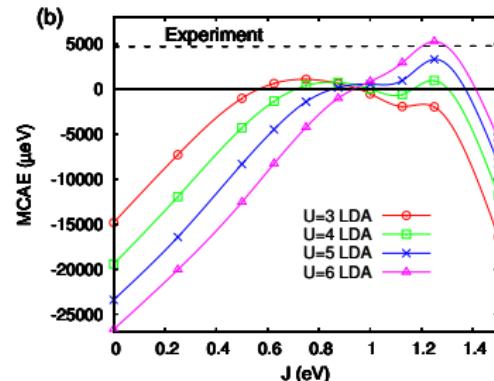
MnF<sub>2</sub>



CoF<sub>2</sub>

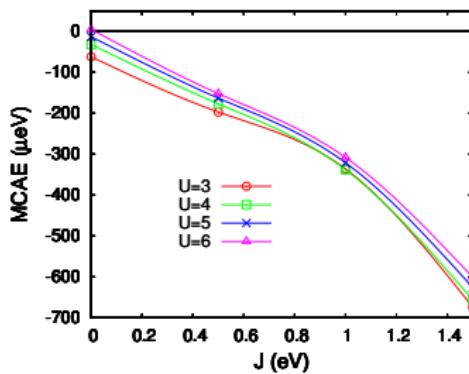


FeF<sub>2</sub>

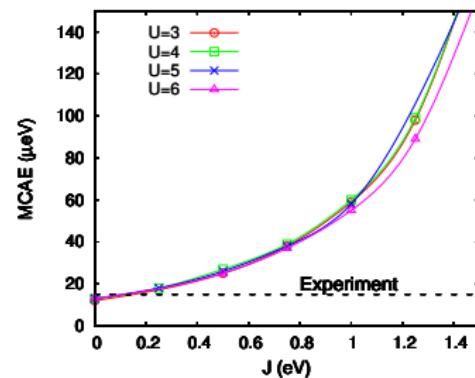


# Magnetocrystalline anisotropy: Difluorites

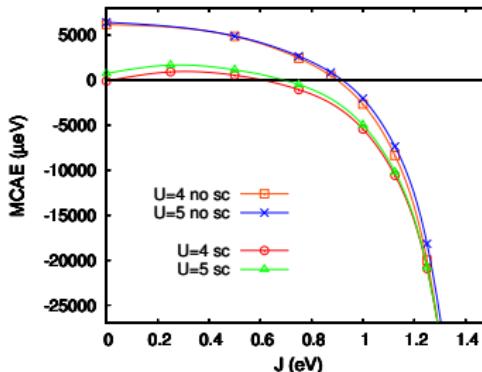
NiF<sub>2</sub>



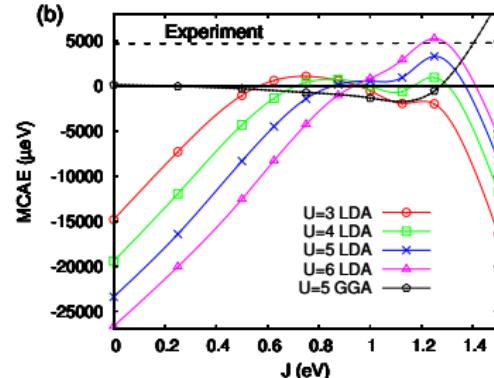
MnF<sub>2</sub>



CoF<sub>2</sub>

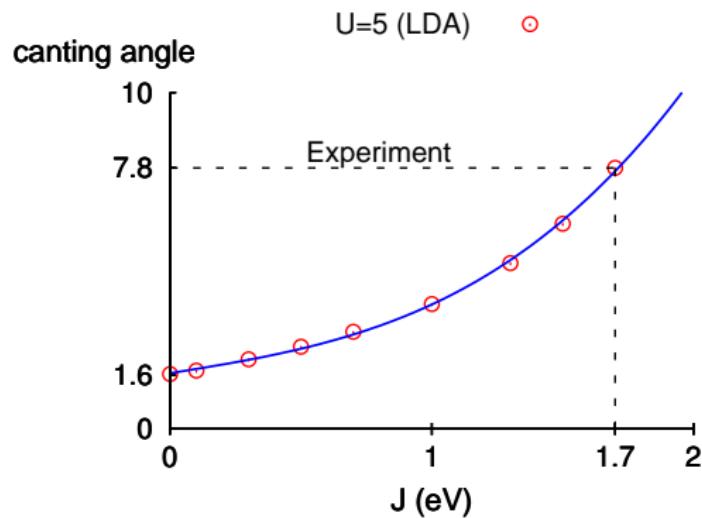


FeF<sub>2</sub>



# Strong $J$ dependence of the canting angle

- LiNiPO<sub>4</sub>: Small effect of  $U$  but canting angle  $\propto J^3$ :



Similar  $J$ -dependence for other systems with spin canting  
(BaNiF<sub>4</sub>, BiFeO<sub>3</sub>, ... )

Bousquet and Spaldin, PRB 82, 220402(R) (2010)

# Origin of the $J$ dependence:

Collinear spins along  $z$ :

$$\rho = \begin{pmatrix} n+m_z & 0 \\ 0 & n-m_z \end{pmatrix} \quad V_{LSDA+U} \equiv \begin{pmatrix} -Un+Jm_z & \times \\ \times & -Un-Jm_z \end{pmatrix}$$

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Collinear spins along  $z$ :

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Spin component along  $x$  (Canting or MCA):

$$\rho = \begin{pmatrix} n+m_z & m_x \\ m_x & n-m_z \end{pmatrix} \quad V_{LSDA+U} \equiv \begin{pmatrix} -Un+Jm_z & Jm_x \\ Jm_x & -Un-Jm_z \end{pmatrix}$$

$J$  acts directly on  $m_x$

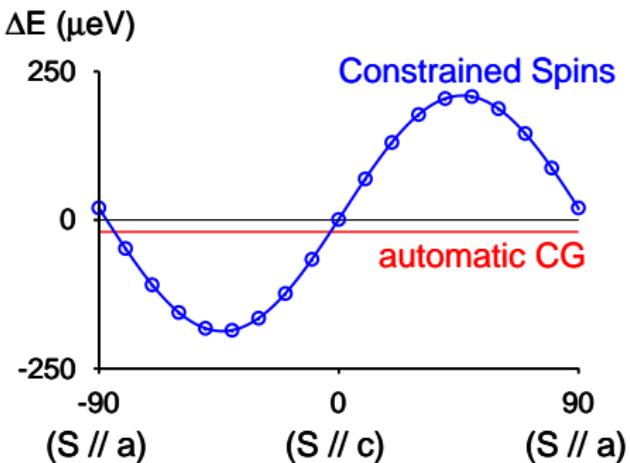
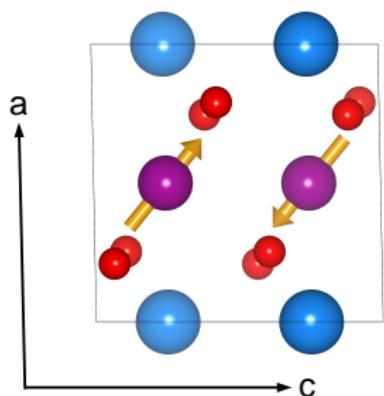
# Beyond LSDA+ $U$

- Problem of predictability of the LSDA+ $U$  with non-collinear magnetism
- Particularly large  $J$  dependence
- Extremely important for MCAE, magnetoelectric response, magnetostriction, piezomagnetism, ...
- Fine tuning of  $U$  and  $J$ : Impossible without experimental measurements!
- Solutions:
  - Self consistent  $U$  and  $J$  ?
  - Hybrids ?
- Challenging case for testing the correctness of new exchange correlation functionals

# Technical problem ...

- GS of non-collinear spins: Global minimum with (a lot of) local minima
- Energy differences: 1–100  $\mu\text{eV}$ !

$\text{MnWO}_4$



- Classical Conjugate Gradient: not trustable!

# In ABINIT?

✓: Working    ✗: Problem    ▷: in process    ✘: Not done

- PAW/NCPSP + non-coll + soc: ✓ (noise on spin orientation ▷ XG, MT, EB)
- LDA+ $U$ : ✓ (need the correct double-counting ▷ BA, EB)
- Constrained magnetic moment: ▷ (I. Lukacevic, MV, EB)
- Finite magnetic field: ▷ (K. Delaney, EB)
- Finite electric field: ✓ ; but:
- Berry phase + non-coll + soc: ✓
- Alternative Algo for SCF: ✘ ?
- DFPT + non-coll + soc: ✘